Middle School Mathematics Institute

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ALLISON COATES, DIRECTOR

Y1 Syllabus

- MONDAY: Formal definitions of Fractions and Decimals, Equivalent Fractions and the Fundamental Fact of Fraction-Pairs (chs. 12 - 13)
- TUESDAY: Equivalent Fractions Contd; Addition of Fractions and Decimals (chs. 14)
- WEDNESDAY: Subtraction of Fractions; Introduction to Multiplication of Fractions (chs. 16 – 17)
- THURSDAY: Multiplication of Fractions Contd.; Division of Fractions (ch. 18)
- FRIDAY: Complex Fractions , Percent , Ratio and Rate (ch. 19 -22)

Daily Schedule

9:00 - 10:30 AM	Lecture
10:45 - 11:30 AM	Discussion
11:30 - 12:30 PM	Lunch
12:30 - 2:00 PM	Lecture
2:15 - 3:30 PM	Discussion
3:30 - 4:30 PM	Q&A

Definitions we have Available

MONDAY morning:

- Defn of fraction: The fraction m/n is defined as: partition the unit segment (or unit square) into n equal length segments (equal areas regions); stack together m such segments (regions), beginning at 0 and moving to the right. The new length (area) is m/n, and is associated with the rightmost endpoint of that length segment.

- Assigned Problems: Chapter 12: 2, 3, 5, 7, 8, 9, 10, 11

Definitions we have Available

MONDAY afternoon:

- Defn of fraction and decimal (a fraction with denominator a power of 10)
- Theorem on Equivalent Fractions: m/n = cm/cm for all fractions m/n and all nonzero whole numbers c.
- FFFP: any two fractions can be denoted by fraction symbols which have the same denominator. For ex, if the given fractions are k/l and m/n (k,l,m,n any whole numbers), then they are respectively also equal to kn/ln and lm/ln.
- Cross Multiplication Algorithm: given two fractions m/n and k/l, m/n = k/l <= => ml = nk

- Assigned Problems: Chapter 13: 1, 3, 4, 5,

Definitions we have available

TUESDAY morning

- Defn of fraction and decimal
- FFFP and CMA
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- Assigned Problems:
  - Chapter 13: 8, 9 , 11
  - Chapter 14: 6, 10,

Definitions we have available

TUESDAY afternoon

- Defn of fraction and decimal
- FFFP and CMA
- Addition of fractions
- k/l of m/n

- Assigned Problems: Chapter 14: 1, 2, 4, 8, 11
- Chapter 15: 2, 4, 8, 9, 10, 12, 15, 16

Definitions we have available

WEDNESDAY morning

- Defn of fraction and decimal
- FFFP and CMA
- k/l of m/n
- Addition and subtraction of fractions

- Assigned Problems: Chapter 16: 1, 2, 5, 9, 12, 15, 17, 19

Definitions we have available

WEDNESDAY afternoon

- Defn of fraction and decimal
- FFFP and CMA
- Addition, subtraction of fractions
- Area Model of multiplication

- Assigned Problems: Chapter 17: 4, 5, 9, 15, 16

CCSSI Standards: Definition of fractions

- GRADE 3: expectations limited to fractions with denoms **2, 3, 4, 6, 8**

1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
    1. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line
    2. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

CCSSI Standards: Definition of fractions

GRADE 4: expectations limited to fractions with denoms 2, 3, 4, 5, 6, 8. 10, 12, 100

- Understand decimal notation for fractions, and compare decimal fractions.
  1. Express a fraction with denominator 10 as an equivalent fraction with denominator 1000, and use this technique to add two fractions with respective denominators 10 and 100.
  2. Use decimal notation for fractions with denominators 10 and 100.
  3. Compare two decimals to hundredths by reasoning about their size. Recognize comparisons are only valid if the two decimals refer to the same whole.

CCSSI: Equivalent fractions

Grade 3: expectations limited to fractions with denoms **2, 3, 4, 6, 8**

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

1. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
2. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent e.g. using a visual fraction model
3. Express whole numbers as fractions, recognize fractions that are equivalent to whole numbers. (3 as 3/1, 5/1 = 5, 4/4 = 1, locate on line)
4. Compare two fractions with the same numerator or same denominator by reasoning about their size. Recognize such comparisons are valid only when the two fractions refer to the same whole. Record the results with symbols, justify conclusions.

CCSSI: Equivalent Fractions

GRADE 4

1. Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

CCSSI: Addition of Fractions

Grade 4:Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

1. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
2. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
3. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
4. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

CCSSI: Addition and Subtraction

GRADE 5:

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

CCSSI: Multiplication

GRADE 4:

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

1. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 * (1/4) recording the conclusion by the equation 5/4 = 5 * ¼.
2. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, 3 * 2/5 = 6 * 1/5.
3. Solve word problems involving multiplication of a fraction by a whole number, e.g. by using visual fraction models and equations to represent the problem.

CCSSI: Multiplication

GRADE 5:

1. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

CCSSI: Multiplication

GRADE 5:

1. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$, *and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$. *(In general,* $(a/b) \times (c/d) = ac/bd$.*)*
2. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

CCSSI: Division

GRADE 6: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.*For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Compute fluently with multi-digit numbers and find common factors and multiples.*

CCSSI: Ratios

GRADE 6: Understand ratio concepts and use ratio reasoning to solve problems.

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, the ratio of wings to beaks in the brid house was 2:1, because for every 2 wings, there was 1 beak.

Understand the concept of a unit rate a/b associated with a ratio a:b with b neq 0, and use rate language in the context of a ratio relationship. For example, this recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.

Use ratio and rate reasoning to solve real world problems…

Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, at that constant rate, how many lawns could be mowed in 35 hours?