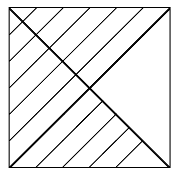
Selected Solutions Allison Coates
Chapter 12, EMI

2. Suppose the unit 1 on the number line is the area of the following shaded region in a division of the given square into four parts of equal area:



   (a) The unit is given as 3 equal-area triangles in the given square. That is, the unit is divided into 3 equal-area triangles. Then, the first picture shows the selected area is 2 of the 3 equal-area triangles. Mapping back to the number line, we've broken the unit into 3 equal length parts, where each part represents 1 of these three equal-area triangles, or $\frac{1}{3}$. Then, two triangles are shaded. So on the number line, we've taken 2 of the three equal length parts. SSo the picture shows 2 copies of $\frac{1}{3}$. Therefore, the fraction represented is $\frac{2}{3}$.

   (b) The second picture shows a shaded region of 3 of 8 equal portions of the large square. The unit is given as 3 of 4 equal area triangles in that large square. To compare the second picture to the reference, we want them to have the same number of equal-area portions. We take the large square and subdivide each triangle again into 2 equal area parts. Now the unit is 6 equal portions, each of area 1/8 of the unit square. The second picture is 3 of those equal portions. Mapping to the number line, we break the unit into 6 parts of equal length, and take 3. Therefore, the second picture represents the fraction 3/6, or 1/2.

   (c) The third picture shows a shaded region of 1 given square (of the same area as our reference) and 1/2 of another (same area) large square. To compare this picture to the unit, we want them to have the same number of equal-area portions. We take the two squares and divide each into 4 equal portions. Now the third picture shows 6 of these equal portions, while the unit shows 3. Mapping to the number line, we divide the unit and each subsequent unit into 3 parts of equal length, and take 6. Therefore, the fraction is 6/3, or 2.

3. The unit 1 is now the 5 shaded equal area regions, each of area 1/8 of the unit square. Mapping to the number line, we break the unit into 5 parts of equal length.

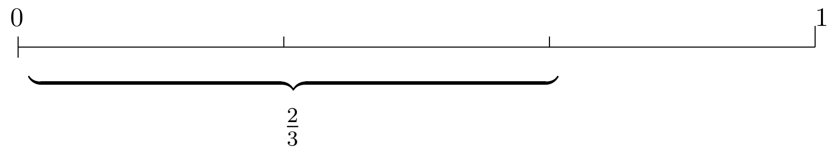
   (a) The picture shows 2 equal sized shaded triangles that are half of the total given square. Represented in rectangles, these 2 triangles would be 4 rectangles, since they are half of the total square, and 4 rectangles are half of the total square. This shaded portion is then 4 rectangles. That is, mapping to the number line, we've broken the unit into 5 parts of equal length and taken 4, or 4/5.

   (b) The picture shows 3 equal rectangles. This shaded portion is then 3 of 5 shaded rectangles, or 3/5.

   (c) The last picture shows 1 large square (with area matching our given square) and 1/2 of another (same area) large square. Drawn with rectangles, this shaded portion is $8 + 4 = 12$ rectangles. Our unit is still 5 rectangles. Represented as a fraction of our unit, this is 12/5.

5. The total unit square is made up of 48×203 congruent rectangles. Congruent rectangles all have the same area. Thus 48×203 the area of each rectangle is equal to 1, the area of the unit square. So the area of any 1 little rectangle must be $\frac{1}{48 \times 203}$.

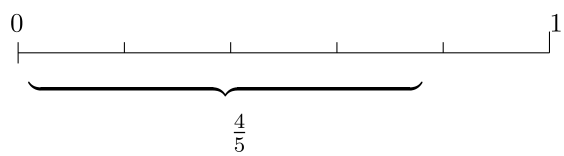
7. (a) After driving 150 miles, we have done only two-thirds of the driving for the day. How many miles did we plan to drive for the day? Explain.



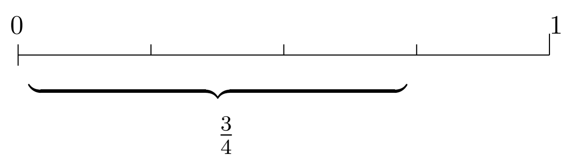
Let the unit be the entire distance we wished to travel that day. 2/3 of that unit means the length found by dividing the unit into 3 parts of equal length and taking two of those parts. Since the length of 2 of the 3 segments of the unit corresponds to driving 150 miles, then the length of one of the segments corresponds to driving 75 miles. We wished to complete $\frac{3}{3}$ segments, or the whole unit. Therefore, we wished to drive $3 * 75$ miles, or 225 miles.

(b) After reading 200 pages, I'm 4/5ths of the way through the book. How many pages are in the book?

The unit is the total number of pages in the book. To define 4/5 of the unit, we partition the unit into 5 segments of equal length, and take 4. This length corresponds to 200 pages. Now, 4 segments corresponds to 200 pages, so one segment corresponds to 50 pages. 5 segments, or the entire unit, is $5 * 50 = 250$ pages.



(c) Helen was 3/4 of the way home, having walked .9 miles.



Recall that by the definition of a decimal, .9 is the same as $\frac{9}{10}$. We define the unit to be Helen's total walk home. To find 3/4 of the walk, we partition the unit into 4 segments of equal length and take 3. The length of 3 such segments corresponds to $\frac{9}{10}$ of a mile. If 3 segments represents $\frac{9}{10}$, then 1 segment is $\frac{3}{10}$. The total length of the unit is then the length of all 4 segments, therefore $\frac{4*3}{10} = \frac{12}{10}$, or 1.2 miles.

8. The distance between 0 and 1 is the same unit distance as the distance between 3 and 4. The pictured left segment goes from the 3 to A. Its length, $\frac{9}{14}$, is $\frac{9}{14}$ths of the length of the unit from 3 to 4. To determine the position of A, we divide each segment between consecutive whole numbers into 14 equal parts. From 0 to 3 we have 3×14 of these parts, and from 3 to A we have another 9. So there are $(3 \times 14) + 9$ of these parts between 0 and A, so $A = \frac{51}{9}$.

The middle segment, from B to 5, is of length $\frac{7}{15}$. It is $\frac{7}{15}$ths of the length of the unit from 4 to 5. To find B, we we divide each segment between consecutive whole numbers into 15 equal parts. From 0 to 4 we have 4×15 of these parts. We break the unit from 4 to 5 into 15 equal segments, and take away the *rightmost* 7. B is then 8 segments from 4, or $(4 \times 15) + 8$ so $B = \frac{68}{15}$.

To find C, we we divide each segment between consecutive whole numbers into 17 equal parts. From 0 to 5 we have $5 \times 17 = 85$ of these parts. We break the unit from 5 to 6 into 17 equal segments as well. The leftmost end of the right segment is 94 segments from 0, and therefore is 9 segments from 5 (since $94 - 85$ is 9), and 8 segments from 6. The right segment is 15 such segments long, so it must extend 7 segments past 6. 6 is $6 \times 17 = 102$ segments from 0. length segments, so the position of C is $\frac{109}{17}$.

9. (a) I earn two dollars for every three times I walk my dog. If I then walk the dog 12 times in a week, how much will I earn?

   Define the unit as the three dog walks, which corresponds to earning 2 dollars.

   Then, to determine the earnings on 12 walks, I must define 12 walks in terms of my unit. As one unit represents 3 walks, 2 units represents 6, 3 units represents 9, and 4 units represents 12 walks. Each unit corresponds to earning 2 dollars, so 4 units corresponds to earning $4 * 2 = 8$ dollars.

   Alternatively, define the unit the number of dog walks in the week, or 12. $12 = 4 * 3$, or 4 times the number of dog walks that earns me 2 dollars. Then, to determine how much is earned, break the unit into 4 parts of equal length, each representing 3 walks. One part corresponds to earning 2 dollars. 4 parts then corresponds to earning $4 * 2 = 8$ dollars.

   (b) Has the fifth grade class been taught a "proportion", let alone "setting up a proportion"? What is a proportion? Is it a fraction? Has the class been taught either about equivalent fractions of the Cross Multiplication Algorithm? (We haven't taught those things yet.) Further, the class is probably not prepared to see the question mark in place of a number in an equality.

   The best way to motivate this is to do the logic as we did it, showing that by reasoning on the number line, we can answer this question–no confusion about proportions, setting them up, or solving them.

10. (a) It is confusing to ask students to create pictures of "equal pieces".What are equal pieces? Equal in number? Equal in shape? Equal in size? Perhaps they think "equal in width" is an acceptable version of "equal pieces". Those are all ambiguous. We must define a unit, and then use that unit: Units are equal in length, or equal in area, etc.

    (b) If we wish them to create pieces of equal areas, we should state so: "construct a picture where the shape is cut into 3 regions of equal area, then shade 2."

11. First, an improper explanation of fractions may lead students to simply be unaware they are being asked to relate the areas of the pattern blocks to each other, especially if up until now, they have used the pattern blocks to explore counting and arithmetic. If a teacher holds up 1 piece and says this is 1, followed by holding up 2 pieces, students may assume she is merely counting objects. Similarly, a student may assume they are discussing some other aspect of the pattern blocks–shape, number of sides, etc. rather than area.

    Instead, she must explain that she is using the pattern block as a unit, and asking about the areas of the blocks in question. "If the area of the yellow hexagon is 1 unit, then how much area of that unit is covered by the area of 2 green triangles?"