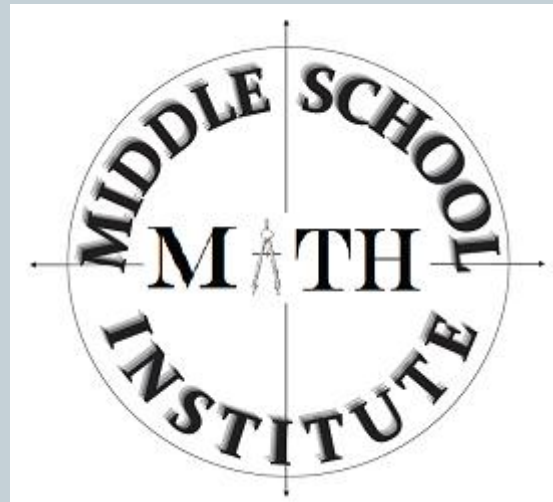


The Math of the SAT: Building Mastery



ALLISON COATES
MIDDLE SCHOOL MATHEMATICS INSTITUTE
WWW.MSMI-MN.ORG



Success in college depends on success in
5th grade math.

Students who are struggling in 5th grade math are knocked off the road to success in college.

Even students **successful in US K-12 math** find it is poor preparation for mathematically heavy majors like engineering, bio, medicine, economics.



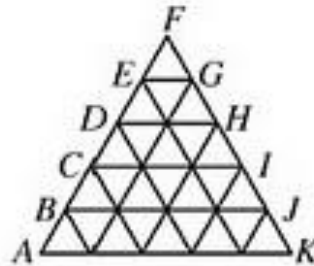
Math portion of the SAT



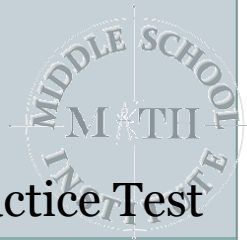
- SAT Reasoning Test (SAT 1) consists of three portions: “Critical Reading, Mathematics, and Writing“
- The **mathematics** section includes questions on arithmetic operations, algebra, geometry, statistics and probability.”
- The SAT is made up of 10 sections,
 - 2 of which are 25-minute mathematics sections
 - 2 of which are 20-minute mathematics sections
- These sections have approximately 16-20 problems each, so a student has approximately 1 minute to solve each problem
including the written answer portion.
- The test scoring was initially scaled to make 500 the mean score on each section with a [standard deviation](#) of 100 (renormed in 1995)



What is mastery?

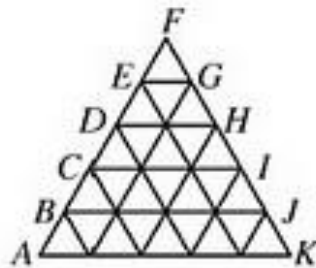


12. The figure above is composed of 25 small triangles that are congruent and equilateral. If the area of $\triangle DFH$ is 10, what is the area of $\triangle AFK$?
- (A) 40
 - (B) 42.5
 - (C) 50
 - (D) 52.5
 - (E) 62.5



Source: College Board, SAT Practice Test

What is mastery?

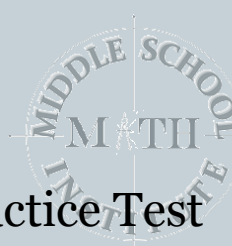


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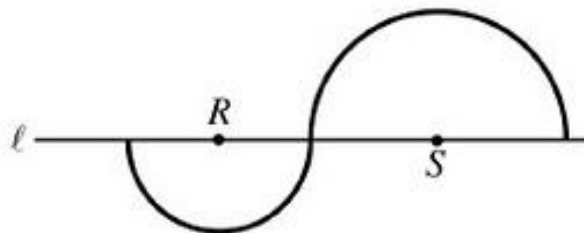
To solve this problem, you must know:

- Congruent
- Equilateral
- Standard defn of triangle
- Relationship of area of similar triangles
- Relationship of Equivalent Fractions/Cross Multiplication Algorithm
- Multiplication and Division of 2 digit numbers



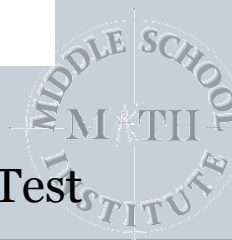
Source: College Board, SAT Practice Test

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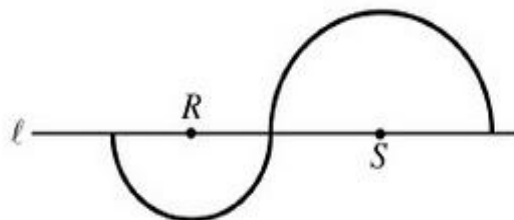


5. The two semicircles in the figure above have centers R and S , respectively. If $RS = 12$, what is the total length of the darkened curve?
- (A) 8π
 - (B) 9π
 - (C) 12π
 - (D) 15π
 - (E) 16π

Source: College Board, SAT Practice Test



What is mastery?

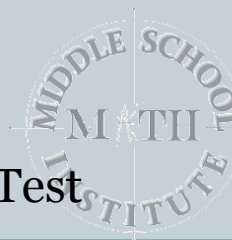


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You must know:

- Defn/circumference of semicircle
- Defn of line's relationship to length
- Arc length of a curve
- Distributive property and immediate ability to use it
- Substitution

Source: College Board, SAT Practice Test



What does mastery require?



MASTERY

**CONCEPTUAL
UNDERSTANDING**

**PROCEDURAL
FLUENCY**



What does mastery require?



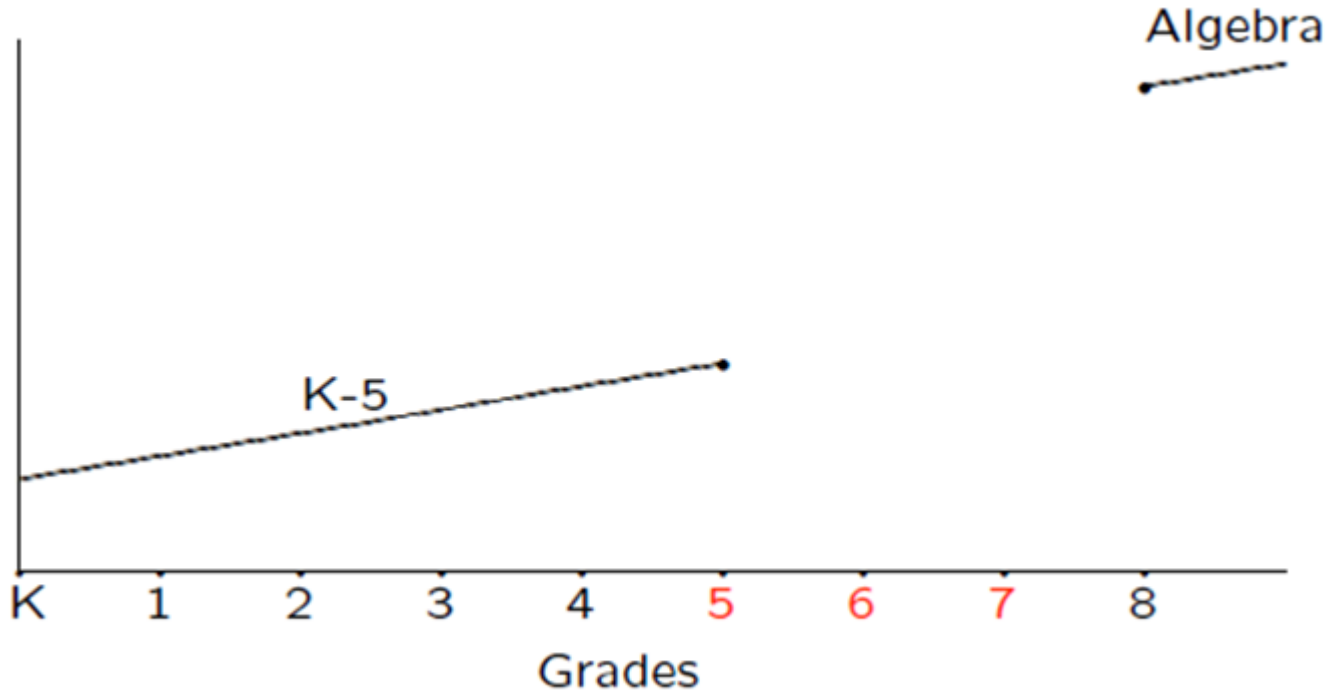
- Some students (and their parents and teachers) **don't notice they don't know *why* they do what they do** in math:
 - no conceptual understanding
- Some students seem to have great conceptual understanding but **we are shocked they can't compute** on their homework:
 - no procedural fluency



Where does the problem begin?

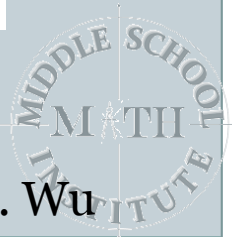


Difficulty



American curricula are exploratory in K-5, while algebra in 8th requires procedural fluency and conceptual understanding of abstractions.

Slide: H. Wu



Shallow Curriculum



Current American curricula/textbooks provide:

- **No Coherence:**
 - They “spiral” shallowly through too many topics, changing subjects often
- **No precision:**
 - Current textbooks do not provide definitions for concepts such as fraction or decimal. Don’t distinguish between definitions and derived results.
- **No Depth:**
 - Textbooks avoid symbolic manipulation. They use manipulatives and artificial examples to avoid abstraction.



Hurting kids with shallow curriculum



- **No Coherence:**
 - Some children learn to hate math because it seems random, illogical, without reason
 - Some children simply get stuck on the procedures because no one can show them how they make sense
- **No precision:**
 - Some children get by with hand waving and coasting. “Math” is easy, until they hit some wall—fractions, algebra, calculus, and then they are totally stuck.



Hurting kids with shallow curriculum



- No Depth:
- Some kids experience class as “too easy” and “too hard” at the same time.
 - “Too easy”: the problems shown in class are mind numbing, hand waving, or off topic (a baking example to teach fractions, endless drawing of “skinnies and bits”, etc.)
 - Yet “too hard”: can’t do 152×183 or $483/15$, or even $342 - 179$. With no practice given to gain procedural fluency, they can’t compute multi-digit arithmetic fast enough to keep up.



Hurting kids with shallow curriculum



- No Depth:
 - Some kids breeze through math classes on their procedural fluency. They crash at high school or college where
 - ✦ They are asked to work hard for the first time
 - ✦ They are asked “why” or how to justify what they are doing
 - ✦ Need to know what their results mean



What preparation do they need?



Place Value

Conceptual Understanding:

10 is ten ones, 100 is ten tens,

16 ounces in a pound, Binary/hexadecimal

Procedural Fluency:

The standard algorithms for addition, subtraction, multiplication, division to automaticity



What preparation do they need?



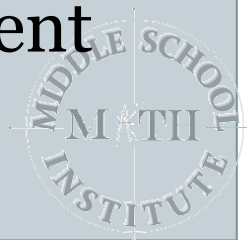
Fractions

Conceptual Understanding:

Fractions as numbers on number line, not pie pieces

Understand WHY for addition you don't add the denominators, but WHY for multiplication you do multiply them

Understand decimals as special case of fractions (with denominator a power of 10), not as different kind of numbers



What preparation do they need?



Fractions

Procedural Fluency

The standard algorithms for addition, subtraction, multiplication, division

Comfortable expressing fractions using definitions:

k/n , for some k , some nonzero n means divide the unit(s) into n equal pieces and take the first k of them

Multi-step word problems by reasoning



What preparation do they need?



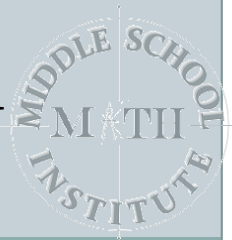
Pre Algebra

Conceptual Development:

What's an equality? **How do I know** when x has a solution? Many? None?

Why am I allowed to manipulate equations in these ways?

Procedural Fluency with Fractions and Standard Algorithms for Arithmetic



What preparation do they need?



- The opportunity to work with abstraction
 - Ex. To use definitions
- The opportunity to work symbolically
 - Ex. Let x be the fraction...
- Practice with mental math and multi-step problem solving
- The opportunity to see that math is coherent



Breadth and Depth



After Stefanie had walked $\frac{2}{5}$ of the distance from home to the train station, there was still $\frac{7}{8}$ of a mile to go. How far is home from the train station? (mid 4th grade, reasoning alone)

A flock of geese on a pond were being observed continuously. At noon, $\frac{1}{5}$ of the geese flew away. At 1 P.M., $\frac{1}{8}$ of the geese that remained flew away. Then 56 geese remained. At no time did any other geese arrive or fly away or die. How many geese were in the original flock? (mid 4th grade, reasoning on number line)

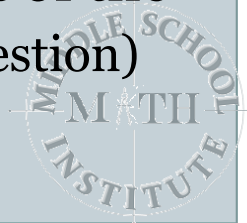


Breadth and Depth



- An alcohol solution mixes 5 parts water with 23 parts alcohol. Then 3 parts water and 14 parts alcohol are added to the solution. Which has a higher concentration of alcohol, the old solution or the new? (a 4-5th grade fraction problem)

A train 132 meter long travels at 87 kilometers per hour and another train 118 meter long travels at 93 kilometers per hour. Both trains are traveling in the same direction on parallel tracks. How many seconds does it take from the time the front of the locomotive of the faster train reaches the end of the slower train to the time that the end of the faster train reaches the front of the locomotive on the slower one? (Sixth-grade Japanese exam question)



More Resources

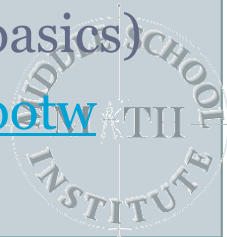


- **Curricular Supplements:**

- Singapore Math's Primary Mathematics, Standards Ed. for gr 1-6:
http://www.singaporemath.com/Primary_Mathematics_Std_s_Ed_s/134.htm
- Japan's textbooks grades 7-9: <http://ucsmp.uchicago.edu/Transl.html>
- Dolciani's Structure and Method for Pre-Alg, Alg 1, Alg 2
http://www.amazon.com/Algebra-Structure-Method-Book-One/dp/0395585309/ref=sr_1_2?ie=UTF8&qid=1299180668&sr=8-2

- **Working Problems:**

- Grades 1-6: Challenging Word Problems for Primary Mathematics 1-6, Singapore Math™ series
http://www.singaporemath.com/Challenging_Word_Problems_s/140.htm
- Grades 5-: The Art of Problem Solving book series (volume 1: basics)
- MATHCOUNTS problem of the week <http://mathcounts.org/potw>



More Resources



- **Teaching:**
 - Parker and Baldrige, Teaching Elementary Mathematics
 - H. Wu, Understanding Numbers in Elementary School Mathematics (available June 2011)



More Resources



- National and Local Math Competitions:
 - MATHCOUNTS:
 - ✦ <http://mathcounts.org> , <http://www.mathcountsmn.org> for grades 6-8 only
 - AMC8 , AMC10, AMC 12: anyone **up to** grades 8, 10, 12 respectively, appropriate for students 3-4 grades below as well
 - ✦ <http://amc.maa.org/e-exams/e4-amc08/amc8.shtml>
 - ✦ <http://www.amc8.org/e-exams/e5-amc10/amc10.shtml>
 - ✦ <http://amc.maa.org/e-exams/e6-amc12/amc12.shtml>
 - Minnesota Junior High Mathematics League
<http://www.usfamily.net/web/mathleague/>
 - Minnesota State High School Mathematics League
<http://www.augsburg.edu/mathleague/>



More Resources



- **Summer and Ongoing Programs in MN:**
 - University of Minnesota Talented Youth Mathematics Program (UMTYMP “um-tee-ump”) for grades 6-12
 - <http://www.mathcep.umn.edu/umtymp/>
 - Institute of Technology Center for Educational Programs for gr 3 - 12
 - <http://mathcep.umn.edu/sumenrich/>
 - Minnesota Institute for Talented Youth for grades 1 -12
 - <http://www.mity.org/>
 - MN State High School Math. League Summer Mathematics Institute (info still to be posted)
 - <http://www.augsburg.edu/mathleague/>
 - Edina Center for Academic Excellence: ongoing Saturday and Summer programs for grades 3 and up: <http://www.ecae.net>



More Resources



- **Summer Camps, Nation wide Programs**
 - Epsilon Camp: for ages 8-11 and their parents!
<http://www.epsiloncamp.org>
 - MathPath for ages 11-14: <http://www.mathpath.org/>
 - IDEA MATH for grades 6-12: <http://www.ideamath.org/>
 - Ross Mathematics Program at Ohio State for ages 14-18:
<http://www.math.osu.edu/ross/>
 - Canada/USA Mathcamp for hs students :
<http://www.mathcamp.org/>
 - PROMYS at Boston U for hs students: <http://www.promys.org>
 - Hampshire College Math Camp for hs students:
<http://www.hcssim.org/>
 - Young Scholars Program in Discrete Math at Rutgers for hs students: <http://dimacs.rutgers.edu/ysp/>

